**Total return swap – Fast Fourier Transformation – Bullet TRS with fixed rate**

**I – Introduction**

This paper aims to highlight Fast Fourier Transformation (FFT) in Total Return Swap (TRS) pricing context. The paper will start by a reminder about FFT methods, classically used for option pricing, this chapter will introduce general ideas which will be the “core” of all papers of FFT module. And, for this paper, a part will be dedicated to model itself for, in first approach, a very simple flavor of TRS.

**II – Reminder of Fourier transformation and TRS generic formula**

Let’s assume a probability density function of a variable X, which distribution has a characteristic function (the gaussian is one of many examples of this kind of law).

The Fourier-cosine approximation of on interval is:

The \* indicates that the first term of the sum should be weighted by 0.5 and indicates the real part of the complex component.

The TRS risk-neutral valuation follows next equations, and at this step we won’t make any assumption about use-case “reduction”, the TRS is assumed to follow one of most complex flavors, like several payments and floating financing rate:

With following notations:

* the TRS flows Pay-off.
* the TRS underlying asset, hidden through variable in integration.
* the interest rate, hidden through variable in integration.
* the bank account.
* the spot underlying price, hidden through the conditional variable in integration.
* the spot interest rate, hidden through the conditional variable in integration.
* the joint probability function for both asset and interest rate.

The Pay-off for TRS flows and bank account are:

All those formulas are generic ones with the interesting two-variable density function, embedding asset and interest risk factors.

**III – TRS pricing, model #1**

For this first model, we took several assumptions:

* TRS has a bullet payment.
* Financing rate is fixed.
* Interest rate is considered as constant.
* Underlying asset follows a lognormal distribution (geometric Brownian motion)
* Formulas will highlight receive performance case, but the algorithm will of course handle both receiver/payer performance total return swaps.

With those previous notes, the previous TRS pricing scheme is heavily simplified and then follows:

With the characteristic function of the normal variable under risk-neutral measure.

For the previous equation, we used the fact that even if indeed the characteristic function of a lognormal distribution can’t be directly calculated, we can use the following “trick”, assuming *:*

And as this time the characteristic function exists and hence we can use the previous result.

Taking following transformation into account:

factors can be derived:

with:

Taking direct variables (instead of highlighting integrals solving under first, the reader can do it if he wants):

We can then rearrange the previous TRs pricing formula:

The algorithm strictly follows previous equations and compare results with direct TRS pricing using risk-neutral, which is here quite trivial as the product is a pure delta one and the discounted asset is a martingale under ℚ measure:

**IV – Model discussion and next axis**

* Of course, the current model is not very useful as the TRS is very vanilla and hence the direct pricing approach is far more useful. But it can be considered as a POC which highlights that FFT also works in TRS context and considered as a “core” for next models, which will be detailed in coming papers.
* Model #2 will relax fixed financing rate assumption, allowing floating interest rate and hence an interest rate distribution, independent from asset distribution.
* Model #3 will do the same, but with joint-distribution asset/interest rate.
* Model #4 will relax the bullet payment assumption, allowing several payments.